

# Transport coefficients of gluonic fluid

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The shear ( $\eta$ ) and bulk ( $\zeta$ ) viscous coefficients have been evaluated for a gluonic fluid. The elastic,  $gg \rightarrow gg$  and the inelastic, number non-conserving,  $gg \rightarrow ggg$  processes have been considered as the dominant perturbative processes in evaluating the viscous co-efficients to entropy density ( $s$ ) ratios. Recently the processes:  $gg \rightarrow ggg$  has been revisited and a correction to the widely used Gunion-Bertsch (GB) formula has been obtained. The  $\eta$  and  $\zeta$  have been evaluated for gluonic fluid with the formula recently derived. At large  $\alpha_s$  the value of  $\eta/s$  approaches its lower bound,  $\sim 1/4\pi$ .

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The nuclear collisions at Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) energies are aimed at creating a phase where the properties of the matter is governed by the quarks and gluons [1], such a phase, which is mainly composed of light quarks and gluons - is called quark gluon plasma (QGP). The weakly interacting picture of the QGP stems from the perception of the QCD asymptotic freedom at high temperatures and densities. However, the experimental data from RHIC, especially the measured elliptic flow [2] indicate that the matter produced at Au+Au collisions exhibit properties which are more like strongly interacting liquid than a weakly interacting gas. The shear viscosity or the internal friction of the fluid symbolizes the ability to transfer momentum over a distance of  $\sim$ mean free path. Therefore, in a system where the constituents interact strongly the transfer of momentum is performed easily - resulting in lower values of  $\eta$ . Consequently such a system may be characterized by a small value of  $\eta/s$ . The importance of viscosity also lies in the fact that it damps out the variation in the velocity and make the fluid flow laminar. A very small viscosity (large Reynold number) may make the flow turbulent.

On the other hand the bulk viscosity exhibit the exchange of energy between the translational and internal degrees of freedom. Although much emphasis has been given to the evaluation of the shear viscosity for a partonic system recently, the bulk viscosity is comparatively less discussed. Probably, because the bulk viscosity for a structureless point particles vanishes both for relativistic and non-relativistic limits [3]. However, there are several reasons for which the bulk viscosity of a system formed in nuclear collisions at ultra-relativistic energies may be non-zero [4]. The trace anomaly in QCD will give rise to non-zero  $\zeta$ , which will indicate the deviation of the system from the conformal invariance, because the  $\zeta$  is defined as the correlation of the trace of the energy momentum tensor through Kubo's formula. The  $\zeta$  for  $SU(3)$  gauge theory has been evaluated in lattice QCD and its value is found to be quite

large around the temperature domain for partons to hadrons transition ( $T_c$ )[5]. The divergence of  $\zeta$  may be treated as a signal of critical point as it diverges near this point [6]. However, this point has been challenged in [7]. This indicate that both  $\eta$  and  $\zeta$  can be used effectively to characterize QGP. Therefore, in the present work we would like to estimate both the ratios,  $\eta/s$  and  $\zeta/s$  for a gluonic fluid by taking into account the perturbative QCD (pQCD) elastic process,  $gg \rightarrow gg$  and the inelastic, number non-conserving process,  $gg \rightarrow ggg$  [8]. While evaluating the transport coefficients we will use the newly obtained matrix element for  $gg \rightarrow ggg$  process [9].

The transport coefficients for QCD matter has been evaluated in [10–12]. The calculation of the viscous coefficients within the ambit of diagrammatic approach of quantum field theory, along with its limitation has been discussed in [13]. Recently pQCD approaches [14–18] have been used to calculate  $\eta/s$ . Evaluation of  $\eta/s$  for a gluonic plasma by Xu and Greiner(XG) indicates that the contribution from  $gg \rightarrow ggg$  is 7 times as large as that from  $gg \rightarrow gg$ . This brings the value of  $\eta/s$  down to the AdS/CFT bound  $\sim 1/4\pi$  [19], when the strong coupling constant,  $\alpha_s = 0.6$ . The GB formula [20](see also [21]) for the  $gg \rightarrow ggg$  matrix element squared is used in [15]. However, we have shown recently that at the lower temperature domain the GB formula receives a significant correction. The ratio of matrix element squared with [9](henceforth will be denoted by the subscript DA) and without [20](henceforth denoted by the subscript GB) the correction term is given by:

$$R_c = \frac{|M_{gg \rightarrow ggg}|_{DA}^2}{|M_{gg \rightarrow ggg}|_{GB}^2} = 1 + \frac{(q_\perp^2 + m_D^2)^2}{s^2} \quad (1)$$

where

$$|M_{gg \rightarrow ggg}|_{DA}^2 = \left( \frac{4g^4 N_c^2 s^2}{N_c^2 - 1 (q_\perp^2 + m_D^2)^2} \right) + \left( \frac{4g^2 N_c q_\perp^2}{k_\perp^2 [(k_\perp - q_\perp)^2 + m_D^2]} \right) + \frac{16g^6 N_c^3}{N_c^2 - 1} \frac{q_\perp^2}{k_\perp^2 [(k_\perp - q_\perp)^2 + m_D^2]} \quad (2)$$

and

$$|M_{gg \rightarrow ggg}|_{GB}^2 = \left( \frac{4g^4 N_c^2 s^2}{N_c^2 - 1 (q_\perp^2 + m_D^2)^2} \right) + \left( \frac{4g^2 N_c q_\perp^2}{k_\perp^2 [(k_\perp - q_\perp)^2 + m_D^2]} \right) \quad (3)$$

where  $g$  is the colour charge,  $N_c$  is the number of the colour,  $m_D \sim gT$  is the thermal (Debye) mass of the gluon,  $k_\perp$  is the transverse momentum of the emitted gluon and  $q_\perp$  is the transverse momentum of the exchanged gluon. Following our previous work [9] we depict the magnitude of the correction,  $R_c$  in Fig. 1. The values of  $q_\perp$  and  $s$  are same as the values taken in Ref. [9]. It is observed that for large values of  $\alpha_s$  the corrections to the GB matrix element is significant. Therefore, it is expected that the values of energy loss,  $\eta/s$  and  $\zeta/s$  will also be affected by the correction term in the lower temperature (higher coupling) domain.

Before discussing the bulk and shear viscosities we estimate the effects of the correction term to the radiative energy loss mechanism of partons propagating through QGP which is measured experimentally thorough the nuclear suppression factor [22] in heavy ion collision. To evaluate the radiative energy loss we start with the soft gluon distribution, which can be written as [9]

$$\frac{dn_g}{d\eta dk_\perp^2} = \frac{C_A \alpha_s}{\pi^2} \left( \frac{q_\perp^2}{k_\perp^2 [(k_\perp - q_\perp)^2 + m_D^2]} \right) + \frac{C_A \alpha_s}{\pi^2} \left( \frac{q_\perp^2 (q_\perp^2 + m_D^2)^2}{s^2 k_\perp^2 [(k_\perp - q_\perp)^2 + m_D^2]} \right) \quad (4)$$

where  $k = (k_0, k_\perp, k_3)$  is the four momenta of the emitted gluon,  $q = (q_0, q_\perp, q_3)$  is the four momenta of the exchanged gluon and  $C_A = 3$  is the Casimir invariant of the  $SU(3)$  adjoint representation. The  $m_D$  in Eq. 3 is the Debye mass required to shield the infra-red divergence. We use the above spectrum of the soft gluon to evaluate the radiative energy loss,  $dE/dx$  of gluons. The minimum value of the momentum of the exchanged gluon in the process:  $gg \rightarrow ggg$  sets time (length) scale for the formation time of the emitted gluon. If the formation time is comparable to or larger than the mean free time (path) then the scattering of the gluons from the successive scatterers in the medium can not be treated as independent

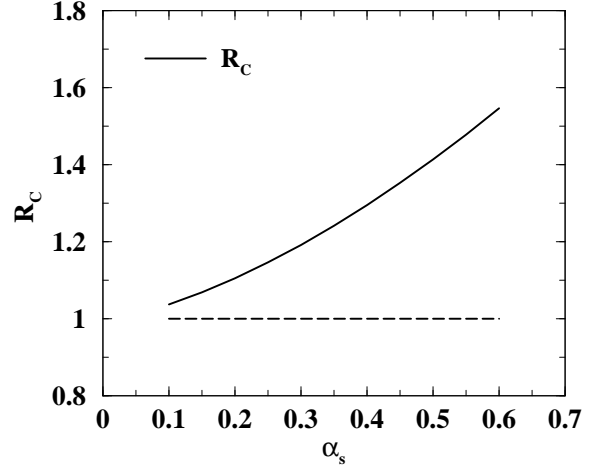


FIG. 1: The variation of the quantity,  $R_c$  (Eq. 1) with coupling constant.

and the scattering amplitudes between two adjacent interaction may interfere destructively leading to the suppression of the emission process - called LPM suppression. The LPM suppression has been taken into account by including a formation time restriction on the phase space of the emitted gluon in which the formation time,  $\tau_F$  must be smaller than the interaction time,  $\tau = \Gamma^{-1}$ ,  $\Gamma^{-1}$  being the interaction rate. The radiative energy loss of heavy quark can be given by [23]:

$$-\frac{dE}{dx} \Big|_{rad} = \Gamma \epsilon = \tau^{-1} \cdot \epsilon \quad (5)$$

where  $\epsilon$ , the average energy per collision is given by

$$\epsilon = \langle n_g k_0 \rangle = \int d\eta d^2 k_\perp \frac{dn_g}{d\eta d^2 k_\perp} k_0 \Theta(\tau - \tau_F) \quad (6)$$

where  $\tau_F = \cosh \eta / k_\perp$ . The  $dE/dx$  is evaluated with temperature dependent  $\alpha_s$ . The variation of  $\alpha_s$  (and hence  $m_D^2 \sim \alpha_s(T)T^2$ ) with  $T$  has been taken from Ref. [24]. In Fig. 2 the variation of radiative energy loss with  $T$  has been depicted for the process  $gg \rightarrow ggg$ . The solid line (dotted line) represents the energy loss when DA (GB) gluon multiplicity distributions are used. To emphasize the importance of the corrections to GB formula we display the ratio,

$$R_{EL} = \frac{DA_{EL}}{GB_{EL}} \quad (7)$$

in the inset of Fig. 2. It is observed that the correction to the gluon spectrum, which leads to the energy loss is appreciable for lower temperature domain. This may affect the suppression of high  $p_T$  partons in QGP and the elliptic flow of the matter formed at RHIC [2] and LHC [25] energies.

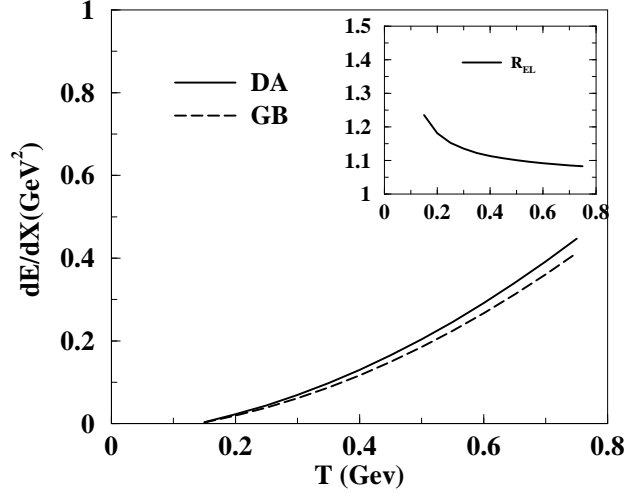


FIG. 2: The variation of energy loss with temperature for the process:  $gg \rightarrow ggg$ . Inset: The Variation of  $R_{EL}$  (Eq. 7) with temperature.

We calculate the quantity,  $\eta/s$  for a gluonic system for the pQCD processes:  $gg \rightarrow gg$ ,  $gg \rightarrow ggg$  and  $ggg \rightarrow gg$ . The  $\eta$  is evaluated using the procedure outlined in [17] (see also [26] for details):

$$\begin{aligned} \eta = & \frac{N_g^2 \beta}{80} \int \prod_{i=1}^4 \frac{d^3 \mathbf{k}_i}{(2\pi)^3 2E_i} |M_{gg \rightarrow gg}|^2 \\ & \times (2\pi)^4 \delta^4(k_1 + k_2 - k_3 - k_4) \\ & \times (1 + f_1^0)(1 + f_2^0) f_3^0 f_4^0 \\ & \times [B_{ij}(k_4) + B_{ij}(k_3) - B_{ij}(k_2) - B_{ij}(k_1)]^2 \\ & + \frac{N_g^2 \beta}{120} \int \prod_{i=1}^5 \frac{d^3 \mathbf{k}_i}{(2\pi)^3 2E_i} |M_{gg \rightarrow ggg}|^2 \\ & \times (2\pi)^4 \delta^4(k_1 + k_2 - k_3 - k_4 - k_5) \\ & \times (1 + f_1^0)(1 + f_2^0) f_3^0 f_4^0 f_5^0 \\ & \times [B_{ij}(k_5) + B_{ij}(k_4) + B_{ij}(k_3) \\ & - B_{ij}(k_2) - B_{ij}(k_1)]^2 \end{aligned} \quad (8)$$

where  $B_{ij}(k) \equiv B(k)(\hat{p}^i \hat{k}^j - \frac{1}{3} \delta_{ij})$ , is defined through the infinitesimal deviation from the equilibrium value of the gluon phase space density. For the process:  $gg \rightarrow ggg$  we use the matrix element obtained recently in [9]. The variation of  $\eta/s$  with  $s = 16 \times 2\pi^2 T^3 / 45$  as a function of  $\alpha_s$  is depicted in Fig. 3. The  $\eta/s$  is quite large at low  $\alpha_s$  because for weakly interacting system the momentum transfer between the constituents become strenuous which give rise to large  $\eta$ . However, with the increase in the coupling strength the momentum transfer gets easier as a result the shear viscosity reduces. The results indicate that for large  $\alpha_s$  the quantity,  $\eta/s$

approaches the AdS/CFT limit. However, in such a scenario the non-perturbative effects may become important. This can be verified by performing a lattice QCD based calculations (which include the non-perturbative effects) for pure SU(3) gauge theory. In fact, such calculation of  $\eta/s$  has been done in [27] and it is found that the value is close to AdS/CFT bound.

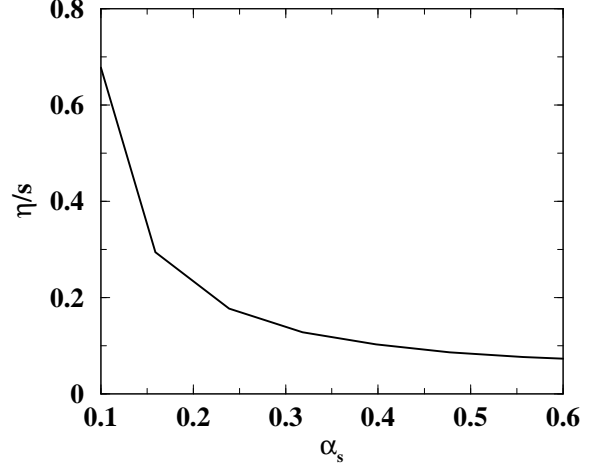


FIG. 3: Variation of  $\eta/s$  with strong coupling constant.

As mentioned before the bulk viscosity, which is connected with the trace of the energy momentum tensor through Kubo's formula, will be non-zero for a system where the conformal symmetry is broken. Lattice QCD calculations indicates non-zero  $\zeta$  for a gluonic plasma [5] due to purely quantum effects (trace anomaly) (see also [28, 29] for QGP and [30] for pions) for temperatures around  $T_c$ . Physically, the bulk viscosity appears in the processes which are accompanied by a change in the volume (i.e. in density) of the fluid. In compression or expansion, as in any rapid change of state, the fluid ceases to be in thermodynamic equilibrium, and internal processes are set up in it which tend to restore the equilibrium. But the processes which drives the system toward equilibrium are irreversible associated with the increase in entropy and therefore involve energy dissipation. Hence, if the relaxation time of these processes is long, a considerable dissipation of energy occurs when the fluid is compressed or expanded and this dissipation must be determined by the bulk viscosity [31]. We evaluate the bulk viscous coefficient with the following formula (see Refs. [32–36]):

$$\zeta = \frac{\text{deg}}{T} \int \frac{p^2 dp}{2\pi^2} \frac{1}{\Gamma(p)} f_p (1 + f_p) [\delta c_s^2 E]^2 \quad (9)$$

where  $\text{deg}$  is the statistical degeneracy for the gluons,  $f_p$  is the Bose-Einstein distribution for the gluons,  $\Gamma$  is the interaction rate, evaluated using the

techniques similar to the one outlined in [17] with the matrix elements  $M_{gg \rightarrow gg}$  and  $M_{gg \rightarrow ggg}$  for the processes  $gg \rightarrow gg$  and  $gg \rightarrow ggg$  respectively,  $c_s^2$  is the velocity of sound and  $\delta c_s^2 = (1/3 - c_s^2)$ . The value of the velocity of sound,  $c_s$  for a massless system in equilibrium is  $1/\sqrt{3}$ , therefore, the results indicate that the bulk viscosity vanishes for a massless system in equilibrium. As  $\delta c_s^2$  is a measure of the deviation from conformal symmetry (for massless system) the  $\zeta$  increases with  $\delta c_s^2$ .

We evaluate the bulk viscosity with the temperature dependent  $c_s^2$  [37]. In Fig. 4, the ratio,  $\zeta/s$  is depicted as a function of strong coupling. The results shown here contain temperature dependence thermal gluonic mass. The most striking observation one can make here is the completely different kind of variation of  $\eta/s$  and  $\zeta/s$  with  $\alpha_s$ . While  $\zeta/s$  increases with  $\alpha_s$  [18], the  $\eta/s$  reduces with it [14]. At small  $\alpha_s$  the bulk viscosity is negligibly small.

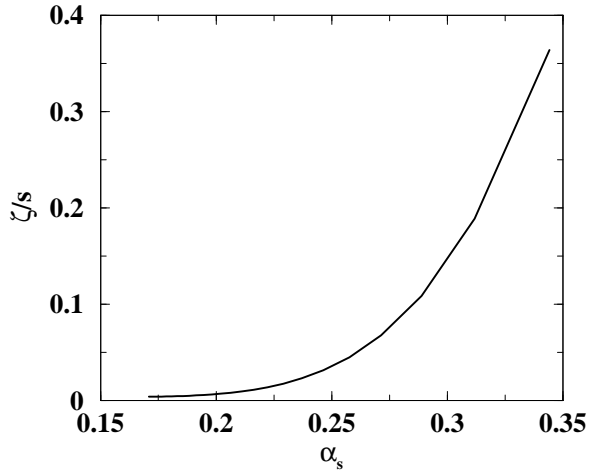


FIG. 4: The variation of  $\zeta/s$  with the strong coupling constant. The temperature dependence gluon mass is included here.

We have evaluated the shear and bulk viscosities for a gluonic system including the pQCD processes:  $gg \rightarrow gg$  and  $gg \rightarrow ggg$ . The matrix element for the later processes is taken from [9]. We find that the value of  $\eta/s$  approaches the AdS/CFT lower bound for large  $\alpha_s$ . The value of  $\eta/s = 0.12$  obtained here at  $\alpha_s = 0.3$  is within the limit extracted from the analysis of elliptic flow of matter formed in nuclear collisions at RHIC energy [38]. The value of  $\zeta/s \sim 0.15$  for  $\alpha_s \sim 0.3$ , which is comparable to  $\eta/s$  at the same value of  $\alpha_s$ .

For  $\alpha_s = 0.47$  where the corrections to GB formula is large, we get the AdS/CFT lower bound  $\eta/s = 1/4\pi$ . Now if we want to understand what is the magnitude of  $\eta/s$  realized in the partonic matter expected to be formed in heavy ion collisions at RHIC and LHC energies, then one possible way is to first estimate the values of temperatures attained in these collisions and then needs to know what is the value of the  $\alpha_s$  corresponding to these temperatures. The variation of strong coupling with temperature may be taken, for example, from [24], for this purpose. The typical values of temperatures which can be achieved in heavy ion collisions at RHIC and LHC energies are  $\sim 300$  MeV and 700 MeV respectively. The magnitudes of  $\alpha_s$  at these temperatures are of the order of 0.23 and 0.17 respectively. From the variation of  $\eta/s$  with  $\alpha_s$  (Fig. 3) we conclude that the typical values of  $\eta/s$  which may realized at RHIC and LHC collisions are 0.177 and 0.29 respectively, which is above the AdS/CFT bound but close to the value obtained from the analysis of experimental data at RHIC [38].

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- [1] J. Alam, S. Chattopadhyay, T. Nayak, B. Sinha and Y. P. Viyogi (ed), J. Phys. G: Nucl. Part. Phys. **35** (2008) (Proc. Quark Matter 2008).
  - [2] I. Arsene *et al.* (BRAHMS Collaboration), Nucl. Phys. A **757**, 1 (2005); B. B. Back *et al.* (PHOBOS Collaboration), Nucl. Phys. A **757**, 28 (2005); J. Adams *et al.* (STAR Collaboration), Nucl. Phys. A **757**, 102 (2005); K. Adcox *et al.* (PHENIX Collaboration), Nucl. Phys. A **757**, 184, (2005).
  - [3] S. Weinberg, Astrophys. J. **168**, 175 (1971).
  - [4] K. Paech and S. Pratt, Phys. Rev. C **74**, 014901 (2006).
  - [5] H. B. Meyer, Phys. Rev. Lett. **100**, 162001 (2008).
  - [6] F. Karsch, D. Kharzeev and K. Tuchin, Phys. Lett. B **663**, 217 (2008); D. Kharzeev and K. Tuchin, JHEP **09**, 093 (2008).
  - [7] G. D. Moore and O. Saremi, JHEP **09**, 015 (2008).
  - [8] F. A. Berends *et al.*, Phys. Lett. B **103**, 124 (1981).
  - [9] S. K. Das and J. Alam, Phys. Rev. D **82**, 051502(R) (2010).
  - [10] A. Hosoya and K. Kajantie, Nucl. Phys. B **250**, 666 (1985).
  - [11] S. Gavin, Nucl. Phys. A **435**, 826 (1985).
  - [12] P. Danielewicz and M. Gyulassy, Phys. Rev. D **31**, 53 (1985).
  - [13] S. Jeon and L. G. Yaffe, Phys. Rev. D **53**, 5799 (1996).
  - [14] P. Arnold, G. D. Moore and L. G. Yaffe, JHEP

- 0305**, 051(2003).
- [15] Z. Xu and C. Greiner, Phys. Rev. Lett **100**, 172301(2008).
  - [16] Z. Xu and C. Greiner and H. Stoecker, Phys. Rev. Lett **101**, 082302(2008).
  - [17] J. W. Chen, H. Dong, K. Ohnishi and Q. Wang, Phys Lett. B **685** 277(2010).
  - [18] P. Arnold, C. Dogan and G. D. Moore, Phys. Rev. D **74**, 085021 (2008).
  - [19] P. Kovtun, D. T. Son and O. A. Starinets, Phys. Rev. Lett. **94**, 111601 (2005).
  - [20] J. F. Gunion and G. Bertsch, Phys. Rev. D **25**, 746(1982).
  - [21] S. M. H. Wong, Nucl. Phys. A **607**, 442 (1996).
  - [22] O. Fochler, Z. Xu and C. Greiner, Phys. Rev. Lett. **102**, 202 (2009); S. Jeon and G. D. Moore, Phys. Rev. C **71**, 034901 (2005); B. G. Zakharov, JETP Lett. **63**, 952 (1996); R. Baier, Y. L. Dokshitzer, A. H. Mueller and D. Schiff, Phys. Rev. C **58**, 1706 (1998); R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff, Nucl. Phys. B **484**, 265 (1997); M. Gyulassy, P. Levai and I. Vitev, Nucl. Phys. B **494**, 371 (2001).
  - [23] S. K Das, J. Alam and P. Mohanty, Phys. Rev. C **82**, 014908(2010).
  - [24] O. Kaczmarek and F. Zantow, Phys. Rev. D, **71**, 114510(2005).
  - [25] K. Aamodt *et al.* (for ALICE collaboration) arXiv:1011.3914.
  - [26] J. W. Chen, J. Deng, H. Dong and Q. Wang Phys. Rev. D **83**, 034031 (2011).
  - [27] H. B. Meyer, Phys. Rev. D **76**, 101701 (2007).
  - [28] S. Datta and S. Gupta, arXiv:1006.0938 [hep-lat].
  - [29] A. Bazavov *et al.*, Phys. Rev. D **80**, 014504 (2009).
  - [30] D. Fernandez-Fraile and A. Gomez Nicola, Phys. Rev. Lett. **102**, 121601 (2009).
  - [31] L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann, Oxford OX2 8DP, UK, 2005.
  - [32] A. S. Khvorostukhin, V. D. Toneev and D. N. Voskresensky, arXiv 1011.0839 [nucl-th].
  - [33] C. Sasaki and K. Redlich, Phys. Rev. C **79**, 055207 (2009).
  - [34] A. S. Khvorostukhin, V. D. Toneev and D. N. Voskresensky, arXiv 0912.2191 [nucl-th].
  - [35] A. S. Khvorostukhin, V. D. Toneev and D. N. Voskresensky, Nucl. Phys. A **845**, 106 (2010).
  - [36] P. Chakraborty and J. I. Kapusta, arXiv:1006.0257 [nucl-th].
  - [37] S. Borsányi *et al.*, arXiv:1007.2580 [hep-lat].
  - [38] R. Averbek, J. Phys. G **35**, 104115 (2008).